STRESSED STATE OF FERROMAGNETIC PLATE WITH CRACKS IN THE PRESENCE OF STRONG MAGNETIC FIELD

L. A. Fil'shtinskii, E. M. Kravets, and V. A. Khvorost

UDC 539.3

If a ferromagnetic body is placed in a magnetic field, body and surface forces appear in the body due to the magnetizability of a material. Under the action of these forces deformations that alter the initial magnetic field appear in the medium.

In the present work we consider a boundary value problem of magnetoelasticity for magnetically soft ferromagnetic medium, weakened by curvilinear cracks. We assume that there is a constant magnetic field $\mathbf{H}_0$ and normal and tangent forces are given on the edges. A similar problem for an isolated straight crack was considered in [1, 2].

1. In linearized statement [1, 3] a complete system of magnetoelasticity relations for magnetically soft piecewise equations of state

$$\mathbf{B} = \mu_0\mu_r\mathbf{H},\quad \mathbf{b} = \mu_0\mu_r\mathbf{h},\quad \mathbf{M} = \chi\mathbf{H},\quad \mathbf{m} = \chi\mathbf{h},$$
$$\mathbf{H} = \mathbf{H}_0 + \mathbf{h},\quad \mathbf{B} = \mathbf{B}_0 + \mathbf{b},\quad \mathbf{M} = \mathbf{M}_0 + \mathbf{m} \tag{1.1}$$

and magnetoelasticity differential equations

$$\operatorname{div} S = 0,\quad \operatorname{rot}\mathbf{H}_0 = 0,\quad \operatorname{div}\mathbf{B}_0 = 0,\quad \operatorname{rot}\mathbf{h} = 0,\quad \operatorname{div}\mathbf{b} = 0,$$
$$S = T + t,\quad t_{ij} = \sigma_{ij} + \mu_0\chi H_{0i}H_{0j} + \mu_0\chi\,(H_{0i}h_j + H_{0j}h_i), \tag{1.2}$$
$$T_{ij} = \mu_0\,(\mu_r H_{0i}H_{0j} - 0{,}5\delta_{ij}H_{0k}H_{0k}) + \mu_0\,[\mu_r\,(H_{0i}h_j + H_{0j}h_i) - \delta_{ij}H_{0k}h_k],$$
$$\sigma_{ij} = \lambda\delta_{ij}\partial_k u_k + \mu\,(\partial_j u_i + \partial_i u_j),\quad \partial_k = \partial/\partial x_k.$$

The boundary conditions in the media boundary line have the form

$$n_1\,(b_1 - b_1^{(e)}) + n_2\,(b_2 - b_2^{(e)}) = \mu_0\chi H_0\cos\psi\cdot n_m\partial u_m/\partial s,$$
$$-n_2\,(h_1 - h_1^{(e)}) + n_1\,(h_2 - h_2^{(e)}) = \chi H_0\sin\psi\cdot n_m\partial u_m/\partial s, \tag{1.3}$$
$$n_1\,(S_{1j} - S_{1j}^{(e)}) + n_2\,(S_{2j} - S_{2j}^{(e)}) = 0.$$

In relations (1.1)-(1.3) $\mu_0 = 4\pi\cdot 10^{-7}$ H/m is the absolute magnetic constant; $\mu_r = 1 + \chi$ is a relative permeability of the medium; $\mathbf{B}_0$, $\mathbf{H}_0$, and $\mathbf{M}_0$ are the magnetic induction, initial magnetic field strength, and magnetization of the material; **b**, **h**, and **m** are fluctuations of these quantities; u_k is the medium elastic displacement; λ and μ are the Lamé constants; δ_{ij} is the Kronecker delta; and ε_{ijk} is the Levi–Civita symbol.

Let an infinite magnetoelastic medium referred to x_1Ox_2 coordinate system be in a strong magnetic field $\mathbf{H}_0 = (0, H_0, 0)$. We assume that there are cracks in the medium, which in undistorted state are associated with mathematical cuts (j = 1, ..., k). On the edges L_j we specify a normal and a tangent load N and T continuously extendible from one edge to the other.

Sumy. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 6, pp. 147-151, November-December, 1993. Original article submitted July 27, 1992; revision submitted January 6, 1993.

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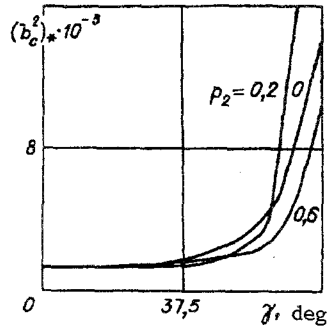

Fig. 1

In this statement the plane problem of magnetoelasticity being considered is reduced to the following system of differential equations:

$$\nabla^2 u_j + \sigma\partial_j\theta + \frac{2}{\mu}\chi\mu_0 H_0\partial_2 h_j = 0 \quad (j = 1, 2),$$
$$\nabla^2 U = 0, \quad h_j = \partial_j U, \quad \nabla^2 = \partial_1^2 + \partial_2^2, \quad \sigma = (1 - 2\nu)^{-1}.$$

The solutions of this system can be expressed in terms of three arbitrary analytic functions by the equations

$$2\mu(u_1 + iu_2) = \varkappa\varphi(z) - zW(z) - \overline{\varphi_1(z)} - C_0 f(z),$$
$$t_{11} + t_{22} = 2\,\mathrm{Re}\,\Phi(z) + 2\,\mathrm{Re}\,W(z) + C_0 H_0,$$
$$t_{22} - t_{11} + 2it_{12} = 2\bar{z}W'(z) + 2W_1(z) + C_0,$$
$$h_1 - ih_2 = iF(z), \quad F(z) = f'(z), \quad \Phi(z) = \varphi'(z),$$
$$\Phi_1(z) = \varphi_1'(z), \quad \varkappa = 3 - 4\nu, \quad W(z) = \Phi(z) - C_0 F(z),$$
$$W_1(z) = \Phi_1(z) - C_0 F(z), \quad C_0 = \chi\mu_0 H_0.$$

The boundary conditions on the cut edges are represented correspondingly. Taking into account (1.3), we can write them in a complex form

$$\begin{aligned} &2\,\mathrm{Re}\,W(z) - \exp(2i\psi)\{\bar{\zeta}W'(\zeta) + W_1(\zeta)\} + C_0\Lambda^*(\zeta) = R(\zeta),\\ &\Lambda^*(\zeta) = \mathrm{Re}\{(1 - i\chi n_2\exp(i\psi))F(\zeta)\},\\ &R(\zeta) = N - iT + C_0 H_0(\chi n_2^2 - 1 + \exp(2i\psi))/2,\\ &n_m\partial u_m/\partial s = \mathrm{Im}\{-\varkappa\Phi(z) - W(z) + C_0 F(z) +\\ &+ \exp(2i\psi)(\bar{z}W'(z) + \Phi_1(z))\}/2\mu, \quad \zeta \in L_j. \end{aligned} \tag{1.4}$$

2. To solve the formulated boundary value problem let us introduce integral representations of the analytic functions:

$$F(z) = \frac{C_0}{2\pi i}\int_L \frac{\omega(\zeta)}{\zeta - z}d\zeta, \quad W(z) = -\frac{1}{2\pi i}\int_L \frac{p(\zeta)}{\zeta - z}d\zeta,$$
$$W_1(z) = \frac{1}{2\pi i}\int_L \frac{\bar{\zeta}p(\zeta)}{(\zeta - z)^2}d\zeta + \frac{1}{2\pi i}\int_L \frac{\overline{q(\zeta)}}{\zeta - z}d\bar{\zeta}.$$

Substitution of limiting values of these functions into boundary conditions (1.4) leads to a system of singular integral equations with respect to displacement jumps and their derivatives:

$$\begin{aligned} &\int_L \{U(\zeta)\,dH_{11}(\zeta, \zeta_0) + V(\zeta)\,dH_{12}(\zeta, \zeta_0)\} = N_1(\zeta_0),\\ &\int_L \{U(\zeta)\,dH_{21}(\zeta, \zeta_0) + V(\zeta)\,dH_{22}(\zeta, \zeta_0)\} = N_2(\zeta_0). \end{aligned} \tag{2.1}$$

Here

$$dH_{11} = \mathrm{Im}\left\{\cdot[-h_1(\psi)(h_4(\psi_0)(\varkappa+1) + h_3(\psi_0)\exp(2i(\psi-\alpha_0))/2) + \right.$$
$$\left. + h_2(\psi)h_3(\psi_0)\exp(2i(\psi-\psi_0))/2 - i\chi^2\mu_0 e_0^2(\varkappa+1)\delta(\psi)h_5(\psi_0)/4]\frac{d\zeta}{\zeta-\zeta_0}\right\};$$
$$dH_{12} = \mathrm{Im}\left\{\cdot[1 + \chi^2(\varkappa+1)ie_0^2 n_1^0 n_2^0/4 + \right.$$
$$\left. + h_3(\psi_0)(\exp(2i(\psi_0-\alpha_0)) - \exp(2i(\psi_0-\psi)))/2]\frac{d\zeta}{\zeta-\zeta_0}\right\};$$
$$dH_{21} = \mathrm{Re}\left\{\cdot[h_6(\psi_0)(h_1(\psi)\exp(2i(\psi_0-\alpha_0)) - h_2(\psi)\exp(2i(\psi_0-\psi)))/2 + \right.$$
$$\left. + \chi(1+\chi\sin^2\psi_0)e_0^2(\varkappa+1)h(\psi,\psi_0)/4]\frac{d\zeta}{\zeta-\zeta_0}\right\};$$
$$dH_{22} = \mathrm{Re}\left\{\cdot[h_6(\psi_0)(\exp(2i(\psi_0-\psi)) - \exp(2i(\psi_0-\alpha_0)))/2 - \right.$$
$$\left. - \chi(\varkappa+1)e_0^2(1+\chi\sin^2\psi_0)/4]\frac{d\zeta}{\zeta-\zeta_0}\right\};$$
$$N_1(\zeta) = (\varkappa+1)\pi\,[N/\mu + \chi e_0^2(\chi n_2^2 + \cos 2\psi - 1)/2]/2;$$
$$N_2(\zeta) = (\varkappa+1)\pi\,[-T/\mu + \chi e_0^2\sin 2\psi/2]/2;$$
$$h_1(\psi) = i + e_0^2\Lambda(\psi);\quad h_2(\psi) = e_0^2\overline{\Lambda_1(\psi)} - i;\quad h_3(\psi) = \frac{\varkappa-1}{\varkappa+1} + 2h_4(\psi);$$
$$h_4(\psi) = \frac{1}{\varkappa+1} + \chi^2 ie_0^2 n_1 n_2/4;$$
$$h_5(\psi) = \overline{\alpha(\psi)} + \chi^2 e_0^2 n_1 n_2(1 - \varkappa + \exp(2i\psi))/2;$$
$$h_6(\psi) = 1 + \chi e_0^2(1+\chi\sin^2\psi)/2;$$
$$h(\psi,\psi_0) = h_1(\psi) + \chi^2\mu_0 e_0^2\delta(\psi)(1 - \varkappa + \exp(2i\psi_0))/2;$$
$$\Lambda(\psi) = \chi\{i + \chi[in_1(2-\varkappa)/\mu_r - n_2(\varkappa+1)]\exp(-i\psi)\}/2;$$
$$\Lambda_1(\psi) = \chi\{-\varkappa i + \chi in_1(2-\varkappa)\exp(-i\psi)/\mu_r\}/2;$$
$$\alpha(\psi) = \chi n_2\exp(-i\psi) - i;$$
$$e_0^2 = \mu_0 H_0^2/\mu;\quad \zeta,\ \zeta_0 \in L_j.$$

System (2.1) should be considered together with the additional conditions

$$\int_{L_j}(U + iV)\,d\zeta = 0\quad (j = \overline{1,k}). \tag{2.2}$$

To solve numerically the system of equations (2.1) and (2.2), it is convenient to carry out parametrization of the contour L_j (below we omit index j): $\zeta = \zeta(\beta)$, $\zeta_0 = \zeta(\beta_0)$, $-1 \le \beta, \beta_0 \le 1$. Following this we represent:

$$U(\zeta) = \frac{\Omega(\beta)}{\sqrt{1-\beta^2}},\quad V(\zeta) = \frac{\Omega^*(\beta)}{\sqrt{1-\beta^2}},\quad a = \zeta(-1),\quad b = \zeta(1).$$

Applying the known quadrature formulas [4], we reduce (2.1) and (2.2) to the system of linear algebraic equations with respect to the values of functions Ω and Ω^* in Chebyshev nodes.

Stress intensity coefficients are determined in the regular way and have the form

$$K_{\mathrm{I}} - iK_{\mathrm{II}} = \pm\mu\sqrt{\overline{s'(\pm 1)}}\left\{-2\frac{\Omega - i\Omega^*}{\varkappa+1} + \chi e_0^2\Omega\left[\frac{\varkappa-1}{\varkappa+1} + \sum_{i,j=1}^{2} a_{ij}^* n_i n_j\right]/2\right\},$$

$$a_{11}^* = -1 - \frac{(2-\varkappa)2\chi}{(\varkappa+1)(\chi+1)},\quad a_{22}^* = -(2\chi+1),$$
$$a_{12}^* = a_{21}^* = i\chi\,[1 - (2-\varkappa)/(\varkappa+1)(\chi+1)].$$

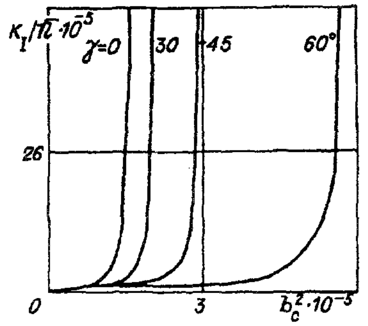

Fig. 2

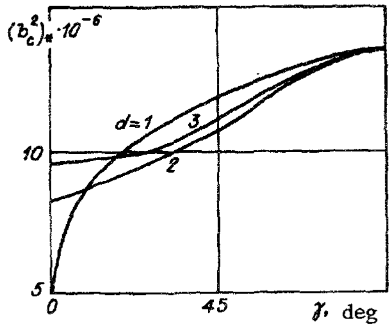

Fig. 3

3. As a first example let us consider the medium weakened by one parabolic crack $x_1 = p_1\beta \cos\gamma - p_2\beta^2 \sin\gamma$, $x_2 = p_1\beta^2 \sin\gamma + p_2\beta^2 \cos\gamma$ ($|\beta| \le 1$) (γ is the angle of crack rotation with respect to the system of coordinates x_1Ox_2). From [1, 2] it follows that there are critical values of the parameter $b_c^2 = b_0^2/\mu_0\mu$, for which the stresses in the medium become infinitely large. For a straight crack, whose front is perpendicular to the direction of the initial magnetic field, $b^2_c = 1.33\cdot 10^{-5}$ (for $\nu = 0.25$ and $\chi = 10^5$). For another crack orientation with respect to the initial magnetic field the critical values will be different. The critical values of b_c2 according to crack orientation γ for different values of the parabolic crack curvature are plotted in Fig. 1. The curves describing the deformation intensity coefficient K_I as a function of the parameter b_c^2 for different orientations of a straight crack are given in Fig. 2.

Let us consider the situation when the medium is weakened by two straight cracks, one of which occupies the interval $[-l; l] = [-1; 1]$ of the x_1-axis and the other is turned to it through the angle γ. The distance between the centers of both cracks is d = 1, 2, 3 and the length of the second crack is $2l_1 = 2$. The critical values of b_c^2 as a function of the angle γ between the cracks, for different distances d between them are plotted in Fig. 3. It is seen that if the second crack is perpendicular to the first one, the value of b_c^2 is the same as for the medium with one crack perpendicular to the initial magnetic field. When both cracks are parallel, b_c^2 increases monotonically with the distance between them.

Analysis of the results shows that the initial magnetic field B_0, whose level is close to the critical value, has a pronounced effect on the equilibrium of ferromagnetic medium with a straight crack. Thus, for a horizontal crack $B_0 \approx 1.2$ T. In the general case critical values of the initial magnetic field levels depend essentially on the mutual arrangement of the defects and their configuration.

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